CONSIDERATIONS

ON THE

BILL

Now depending in the

HOUSE of COMMONS,

For enabling Pariſhes to grant Life-Annuities to Poor Perſons, upon Purchaſe, in certain Circumſtances, and under certain Reſtrictions.

BEING

An APPENDIX to the Pamphlet, intitled, "A PROPOSAL for Eſtabliſhing Life-Annuities in Pariſhes for the Benefit of the Induſtrious Poor."

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Conſiderations on the Bill now depending in the Houſe of Commons for enabling Pariſhes to grant Life-Annuities to Poor Perſons, upon Purchaſe, in certain Circumſtances, and under certain Reſtrictions.

Being an APPENDIX to the Pamphlet, intitled, "A PROPOSAL for Eſtabliſhing Life-Annuities in Pariſhes for the Benefit of the Induſtrious Poor."

IN order to make the grounds and reaſons, upon which the values of the life-annuities recommended in the foregoing propoſal are to be computed, as plain and familiar as poſſible, and to remove all doubts concerning the truth and juſtneſs of them, I ſhall here ſubjoin the ſolution of a ſecond problem, which is the *converſe* of that which is ſolved in the 15th, 16th, 17th, and 18th pages of the foregoing pamphlet. In that problem it was required to determine what life-annuity, to commence at a remote future period, a man of a given age could purchaſe with a given

a given ſum of money. In this it is required to determine what ſum of money a man of a given age ought to pay as the price of a given life-annuity, to commence at a remote future period. In both problems the age of the purchaſer of the annuity is ſuppoſed to be 25, and the annuity is ſuppoſed to commence at the end of 25 years, or when he is 50 years of age, ſo that the firſt year's payment of it ſhall be made when he is 51; and the intereſt of money is ſuppoſed to be 3 per cent, and the probabilities of the duration of human life at different ages to be ſuch as they are repreſented to be in the table given of them in Mr. Simpſon's doctrine of annuities and reverſions, page 4. This ſecond problem is as follows.

PROBLEM II.

Suppoſe a man of 25 years of age ſhould be willing to purchaſe an annuity of one pound a year for his life, to commence when he ſhall be 50 years old, that is, ſo that the firſt year's payment ſhall be made to him when he is exactly 51 years old. It is required to determine what price he ought to pay for ſuch an annuity; upon a ſup-

a ſuppoſition that the intereſt of money is 3 per cent, and that the probabilities of the duration of human life at different ages are ſuch as they are repreſented to be in Mr. Simpſon's doctrine of annuities, page 4, and that the values of life-annuities that are to commence immediately, or whereof the firſt payments are to be made at the end of a year from the times at which they are purchaſed, are ſuch as they are repreſented to be in the ſame treatiſe of Mr. Simpſon, page 39.

Now it appears by Mr. Simpſon's table of the values of life-annuities in the treatiſe aforeſaid, page 39, that, if the purchaſer of the annuity of 1l. a year was already 50 years old, the price he ought to pay for it would be 11.4l. or 11l. 8s. But as he is ſuppoſed to be only 25 years old, and may perhaps never live to be 50 years of age, and, if he ſhould, the money he will have paid down at 25 for this annuity will have increaſed in the hands of the grantor of it, to ſome greater ſum, during thoſe 25 years, by being improved at compound intereſt at the rate of 3 per cent, it is evident that on both theſe accounts the ſum he ought to pay for this

remote

remote annuity will be leſs than 11l. 8s. or 11.4l.

In what proportion it will be leſs than that ſum is what we are next to determine.

Now as there are two grounds for diminiſhing this ſum of 11.4l. which are totally ſeparate and independent of each other, it will be proper to conſider them ſeparately.

In the firſt place therefore let us ſuppoſe either that the purchaſer of the annuity was abſolutely ſure of living five and twenty years longer, or till he was 50 years of age, or elſe (which amounts to the ſame thing) that the grantor of the annuity had agreed that, if he died before that time, the grantor ſhould pay to his executor at the end of thoſe five and twenty years, the ſum of 11.4l. or 11l. 8s. which is the value which the annuity would have borne at that time if he had lived to become intitled to it. In this caſe it is evident that the purchaſer of the annuity ought to pay to the grantor of it ſuch a ſum of money as being improved by the grantor at compound intereſt at the rate of 3 per cent during the ſpace of 25 years, will

amount

amount at the end of that time to the ſaid ſum of 11.4l. For then the grantor will juſt be enabled thereby, in caſe of the death of the purchaſer, to pay to his executor the ſaid ſum of 11.4l. at the end of the ſaid 25 years according to the agreement; and ſo will be neither a gainer nor a loſer by his bargain: and in the other event, of the purchaſer's living to the age of 50, the grantor will have received at that time, by means of the payment made to him by the purchaſer when he was 25 years old, and the improvement he will have made of it during thoſe 25 years, juſt the very ſum of 11.4l. which he ought to receive as a fair equivalent for his annuity of one pound a year for the life of a man of 50. We muſt therefore enquire what ſum of money, being improved at compound intereſt, at the rate of 3 per cent, for the ſpace of 25 years, will at the end of that time amount to 11.4l. Now, by the table in page 112 of Mr. Simpſon's Doctrine of annuities it appears that the preſent value of 1l. to be received at the end of 25 years, if the intereſt of money is 3 per cent, is .4776 of a pound; or that .4776, or $\frac{4776}{10000}$, of a pound, will, if improved at compound intereſt, grow

up

up in that number of years to the ſum of one pound. Therefore 11.4 times .4776 of a pound, will, if improved in the ſame manner during the ſame time, grow up to 11.4 times one pound, or 11.4l; that is, 5.44464l, will, if improved in that manner for the ſpace of 25 years, become equal to 11.4l. or 11l. 8s. Therefore if the purchaſer aforeſaid of the annuity of one pound, to commence when he is 50 years of age, was ſure of living to that age, or if the grantor had agreed, in caſe of his dying before that time, to pay to his executor at the end of the ſaid 25 years the ſum of 11l. 8s. the ſum which the ſaid purchaſer ought to pay for ſuch an annuity at the age of 25, would be 5.44464l. or 5l. 8s. 10½d.

But in the preſent queſtion it is ſuppoſed that, in caſe the purchaſer of the annuity dies before he attains the age of fifty, the grantor is to keep the whole purchaſe-money without making any payment whatſoever to his executor. Conſequently a leſs ſum than 5l. 8s. 10d. ½ or 5.44464l. ought to be paid by the purchaſer as the price of this annuity. For otherwiſe the bargain would be too favourable to the grantor; ſince he would receive the ſame

price

price for the annuity when not accompanied with the condition of paying to the purchaſer's executor the ſum of 11.4l. at the end of 25 years, in caſe of the death of the purchaſer before that time, as was a fair price for it, when encumbered with that condition. We muſt therefore in the laſt place endeavour to find out how much the ſum of 5l. 8s. 10d. halfpenny, or 5.444641. ought to be diminiſhed on account of the removal of this condition, or on account of the chance which the grantor has, on the terms of the preſent queſtion, of gaining the whole purchaſe-money for nothing, by means of the death of the purchaſer, in the courſe of the next twenty-five years.

Now, in order to eſtimate this chance, we muſt ſuppoſe the grantor to make a great number of grants of ſuch annuities to ſeveral perſons of the ſame age of twenty-five years, and of the ſame degree of health and ſtrength, and placed in the ſame circumſtances of danger or ſafety to their lives, and in all reſpects equally likely to live to any given future age, upon the ſame terms, and to commence at the ſame time, to wit, when they ſhall be fifty years of age. And

we muſt determine what price each of theſe many purchaſers ought to pay to the grantor for their ſeveral annuities of 1l. ſo as to produce a moral certainty that the grantor will be neither a loſer nor a gainer by the ſum total of all his grants. And this will be the fair price which a ſingle purchaſer ought to pay him for his annuity.

Now, in order to make the grantor of theſe ſeveral remote annuities of 1l. each, to ſeveral different purchaſers all of the age of twenty-five, be neither a gainer nor a loſer by the ſum total of all his grants, it is evident that the prices paid for theſe annuities by the ſeveral purchaſers of them ought to be ſuch that, if they are improved at compound intereſt at the rate of 3 per cent for 25 years, they will, together, produce ſo many ſums of 111l. 8s. or 11.41. as there will be perſons out of the whole number of purchaſers that will live to be fifty years old; becauſe that is the juſt value of the life-annuity to which each of them will then become intitled. Now it appears by Simpſon's doctrine of annuities, page 4, that of 426 perſons of twenty-five years of age, living at the

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ſame time, only 204 will attain the age of fifty. Therefore, if we ſuppoſe the whole number of perſons who have purchaſed theſe annuities to be 426, the ſums contributed by them muſt be ſuch that, if they are improved at compound intereſt, at the rate of 3 per cent, for twenty-five years, they will, at the end of that time, produce 204 times 11l. 8s. or 204 times 11.41. that is, 2325.61. or 2325l. 12s. Therefore the ſum to be paid by each of the ſaid 426 purchaſers for his annuity muſt be ſuch as, being improved at compound intereſt at the rate of 3 per cent for the ſpace of twenty-five years, will, at the end of that time, be equal to the 426th part of 2325.61. that is, to 5.4591. or 5l. 9s. 2d. Now, ſince .47761. or $\frac{4776}{10000}$ of a pound, being improved at compound intereſt, at the rate of 3 per cent, for twenty-five years, will, at the end of that time, be equal to a whole pound, it is evident that 5.459 times .47761. or 2.6071. or 2l. 12s. 1d. halfpenny, being improved in the ſame manner during the ſame time, will produce 5.459 times a whole pound, or the ſum of 5.4591. or 5l. 9s. 2d. Therefore 2l. 12s. 1d. halfpenny is the ſum which each of the 426 purchaſers of theſe an-

nuities ought to pay to the grantor for his annuity of one pound per annum, to commence when he is 50 years of age. Therefore if only one ſuch annuity is granted to a perſon of twenty-five years of age, of an ordinary degree of health, and whoſe chance of living to any given future age is neither greater nor leſs than that of the common run of men, to commence when he ſhall have attained the age of fifty years, the price he ought to pay for it is 2l. 12s. 1d. halfpenny.

Coroll. Let the annuity to be purchaſed by the perſon of twenty-five, to commence at the age of fifty, be 3l. 16s. 8d. halfpenny, or 3.835l. inſtead of 1l. Then ſince 2l. 12s. 1d. halfpenny, or 2.607l. is the price of an annuity of 1l. to commence at that age, the price of an annuity of 3.835l. to commence at the ſame age, will be 3.835 times 2.607l. that is, 9.997l. or 10l; which agrees with the concluſion given in the foregoing pamphlet, page 18.

January 18, 1773.

WITHIN theſe few days I have been favoured with the ſight of the following objections to the bill now depending in the Houſe of Commons for enabling pariſhes to grant life-annuities of the kind above-mentioned to ſuch poor inhabitants as ſhall be diſpoſed to purchaſe them. This bill is conſiderably different from the draught propoſed at the end of the foregoing pamphlet, which has been carefully examined and greatly improved by ſeveral gentlemen of abilities who think the ſcheme may be of public benefit. The objections, with the anſwers which it ſeems reaſonable to give them reſpectively, are as follows:

OBJECTIONS to the Annuity-bill for Poor Perſons, with the Anſwers to them.

OBJECTION I.

It will check matrimony.

ANSWER.

I cannot conceive how it will have this effect. When two perſons that are diſpoſed to marry each

each other, have each an annuity of twenty pounds a year for their life to commence when they ſhall be fifty, or ſixty years of age, which they have purchaſed with the money ſaved out of their wages, or other daily gettings, and conſequently are in a habit of living upon leſs than they uſually acquire by their induſtry, it ſhould ſeem that they would have nothing to hinder them from entering into the married ſtate, and that the proſpect of enjoying thoſe annuities in their old age would make them leſs fearful of engaging in it than if they had no ſuch proviſion. The poſſeſſion of property, though but for life, and even a reverſionary right to ſuch property, is generally found to operate as an encouragement to matrimony. A young clergyman that has a good living in the church, or is intitled to one upon the next vacancy, is more likely to marry than a poor curate, or even than another clergyman who ſhould be poſſeſſed of a ſum of money juſt ſufficient to purchaſe ſuch a living.

OBJECTION II.

It will put too early a period to the poor man's labour.

ANSWER.

ANSWER.

It is probable that theſe annuities will be bought only by houſhold ſervants in gentlemen's, or rich tradeſmen's, families, and by journeymen manufacturers and handycraftsmen; and not by day-labourers in huſbandry, who are hardly able to ſave any thing out of their wages. Now theſe are the perſons who ſeem to be meant in the objection. And therefore it will not put any period at all to their labour, as is ſuppoſed.

Further, the perſons who will buy thoſe annuities out of the ſavings made upon their daily earnings, will be probably of ſo induſtrious a diſpoſition that they will not be induced to leave their employments, and live idle, by the poſſeſſion of the greateſt annuity that can be purchaſed under this act of parliament, which is but 20l. a year; but will rather endeavour to mend their ſituation ſtill further, and in other reſpects, and to advance themſelves to a higher ſtation in life by keeping ſhops, or becoming maſters in their ſeveral handicraft trades. The poſſeſſion of a little property acquired by induſtry is generally an incitement to get more.

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And when theſe perſons had ſecured themſelves againſt the danger of diſtreſs in their old age by employing ſome of their firſt ſavings in the purchaſe of theſe annuities, their next ſavings would be employed in advancing themſelves in the world in ſome other way, ſuch as the purchaſe of a houſe, or a bit of land, or ſome ſtock in the public funds, or the furniture of a houſe, or a ſhop, or the like.

OBJECTION III.

It will increaſe the grievance now ſo much complained of, of throwing farms together; as the majority of little farmers grow out of day-labourers, and often to men of conſequence; whereas, by turning their ſavings into an annuity, their ambition is bounded.

ANSWER.

The throwing farms together is almoſt always owing to the pleaſure of the land-lord; who does it, partly to get rid of all the expence of repairing a number of farm-houſes; partly to leſſen the number of people in the pariſh, and thereby diminiſh the poor's rate; and partly to facilitate and ſecure the receipt of his rent

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by having to do with only one or two rich, or ſubſtantial, farmers, inſtead of a dozen ſmall ones, who may be hardly able to pay their rents. There never is a want of perſons ready to take ſmall farms; but always, as I apprehend, many more than enough for this purpoſe: and therefore there is no danger of increaſing the practice of throwing the ſmall farms together by diverting the money ſaved by day-labourers into another channel by offering them life-annuities. But in truth I fear, as is above mentioned, that very few day-labourers will be able to ſave money enough out of their ſcanty wages to purchaſe theſe life-annuities.

OBJECTION IV.

If the annuity purchaſed be but ſmall, not much beyond the pariſh-allowance, will not the purchaſer be perpetually preſſed by neceſſities and the importunities of his family, or the ridicule of his idler neighbours, to part with it, and throw himſelf on the pariſh? they telling him he will be full as well off then as now.

ANSWER.

I do not ſuppoſe that he will be induced to part with it upon theſe motives, as he will feel the pleaſure of having ſomething of his own, independent of the charity of the pariſh. But, if he ſhould, he will, after he ſhall have parted with it, be no worſe off than if he had never bought it, or than if this act had never paſſed: and in the mean time he will have been benefited by the act by having been excited thereby to a greater degree of induſtry in order to get and ſave the money with which he bought it. So that this act will, upon the whole, have done *no* harm, and *ſome* good, even in this caſe.

If the annuity be ſmall, and little better than the pariſh allowance, which is often not more than a ſhilling a week, or 2l. 12s. a year, I conceive that the perſon who ſhall have purchaſed it, will, inſtead of parting with it, continue in his courſe of induſtry and frugality, in order to ſave money enough to purchaſe an additional annuity that ſhall make it a more comfortable proviſion; and even, if he

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can poſſibly ſave money enough for the purpoſe, he will be anxious to increaſe it to 20l. a year, which is the greateſt annuity the act allows. And this is a very different proviſion from the mere pariſh allowance of 2l. 12s. or 3l. or even 5l. 4s. a year, or 2s. a week, which is all that is allowed in the rich pariſh of St. James in London.

OBJECTION V.

And, if the annuity be unalienable, will it not be very ill-judged to purchaſe it? as any little alteration in the annuitant's circumſtances may make the money of much more conſequence to him.

ANSWER.

The annuity is not to be unalienable, unleſs the purchaſer expreſsly conſents that it ſhall be ſo, and this conſent is expreſſed in a clauſe for that purpoſe inſerted in the deed of grant. Now it may reaſonably be ſuppoſed, that, when a man purchaſes one of theſe annuities with his own money, he will not conſent to ſuch a clauſe of unalienability, but will rather re-

ſerve to himſelf the power of parting with it in caſe he ſhall be ſo inclined. And this clauſe of unalienability will probably never be inſerted in the grants of theſe annuities, unleſs when they are bought for the grantees of them by ſome charitable friend, who may inſiſt on their conſenting to this clauſe, that they may not, through their ſubſequent extravagance and indiſcretion, fruſtrate his good intentions in their favour by parting with the proviſion he has bought for them.

OBJECTION VI.

May it not prevent young people from beating out from home to ſeek their livelyhoods? which is perpetually done to their infinite advantage.

ANSWER.

I cannot ſee how it will have this effect. It will be neceſſary for a young man to exert a good deal of induſtry to acquire the money neceſſary to purchaſe an annuity of 20l. to commence when he is fifty years old. This induſtry he muſt exert either at home or abroad;

abroad; and he will probably do it in the one place or the other, as he ſhall find his opportunities invite him. And when he has got the money, and bought the annuity, or had it bought for him, yet ſtill he will be under a neceſſity of exerciſing his induſtry ſomewhere till the annuity becomes due, which will not be till he is fifty years old. In the mean while, (which takes in all the vigour of his life) he is at liberty to beat out, as the objection expreſſes it, wherever he can, and has as much reaſon to do ſo as if he was not intitled to the annuity. For how can the expectation of an annuity of 20l. a year, to take place at fifty, or fifty-five, years of age, hinder a man of twenty-five, or thirty, years of age, from going any where, by ſea or land, to ſeek his livelyhood, and improve his fortune, in the mean time?

OBJECTION VII.

Are not annuitants, at beſt, reckoned but drones in every ſtate?

ANSWER.

Thoſe who are poſſeſſed of life-annuities ought no more to be reckoned drones in a ſtate

ſtate than thoſe who live upon any other certain income, not ariſing from their induſtry, ſuch as the rent of a houſe, or a farm, or the intereſt of money in the publick funds. In all theſe caſes they live upon an income ariſing from the labour of others: yet are not reckoned drones, or uſeleſs members of ſociety, on this account, unleſs they live abſolutely idle, and follow nothing but their own pleaſure, without conſulting the good of others. It is this, and not the nature of the income on which he lives, that denominates a man a drone. Thus a man of 3000l. a year in land, that lives conſtantly in a ſtate of celibacy either at London, or ſome public place of pleaſurable reſort at a diſtance from his eſtate, and ſpends his time in eating and drinking, dreſſing and card-playing, and going to places of public entertainment, ſuch as plays, operas, concerts, maſquerades, and the like, may juſtly be reckoned a drone, though his income ariſes from freehold land of inheritance. For he reaps where another ſows, that is, he lives upon a part of the product of his tenants labour, and with a view merely to his own pleaſure, without making himſelf uſeful to others. And

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on the other hand, a man poſſeſſed of the ſame income of 3000l. a year, ariſing from a life-annuity, or great ſinecure place, that is a diligent and upright member of parliament, or an uſeful juſtice of peace, or a faithful manager of the revenue of ſome public hoſpital, or other uſeful inſtitution, or of the affairs of ſome great trading company, or that diligently cultivates ſome uſeful branch of learning or ſcience, (which only men of fortune are at leiſure to cultivate) or is a married man with ſeveral children, and is very careful of their morals and education; I ſay, a man that does ſeveral, or even any, of theſe things, will never be reckoned a drone, notwithſtanding he is poſſeſſed of an annuity for his life and lives upon nothing elſe. And the ſame may be ſaid concerning ſmaller incomes, and lower ſtations in life. Now, with reſpect to theſe pariſh-annuities, as they are reſtrained to 20l. a year to each purchaſer, it is preſumed they will not be ſufficient to make drones of thoſe who ſhall have bought them, even when they are come into the poſſeſſion of them; and till then they cannot have any tendency that way, ſince the owners of them will reap no advantage from them,

them, but be under the ſame neceſſity of working for their livelyhood as if they had not bought them; or rather under a ſtill greater neceſſity of doing ſo, becauſe they will have parted with that money for the purchaſe of them which they might otherwiſe ſpend in ſupporting themſelves in preſent idleneſs for ſome ſhort time. And they will evidently have operated in a way directly oppoſite to that of making them drones, during all the time preceding the purchaſe of them; namely, as an additional incentive to their induſtry, that they might be able to earn and ſave enough money to make the ſaid purchaſe. Thus it appears that both before the purchaſe of theſe annuities and after it till the purchaſers come into the actual receipt of them, that is, till the men are at leaſt fifty, and the women at leaſt thirty-five, years old, the purchaſe of theſe annuities will operate as a ſtrong additional incentive to the induſtry of the perſons who purchaſe them; which will be a great public advantage, (ſince that is the time of their youth and ſtrength) even though we ſhould ſuppoſe that they would afterwards live idle upon them. But this is a ſuppoſition I can by no means think

think probable: ſince perſons who have perſevered in a courſe of induſtry to the age of fifty, have uſually ſuch a taſte and liking for it, and for the advantage it brings with it, that they are in no danger of forſaking it.

Further it may be obſerved that, though the annuities may commence at the age of fifty, yet it is probable they will not often commence before ſixty; becauſe the great difference between the annuity which a man of twenty-five or thirty years of age, and ſtill more one of thirty-five or forty years, (at ſome of which ages we may ſuppoſe moſt of the annuities to be purchaſed) can purchaſe for a given ſum of money, as for inſtance, 10 pounds, to commence when he ſhall be fifty years of age, and that which he can buy for the ſame money, to commence when he is ſixty years of age, will probably induce him to buy it for the latter age, till which time he may very reaſonably hope he ſhall be able to maintain himſelf with eaſe, by his own induſtry. So that I reckon that moſt of the annuities will be purchaſed to commence at ſixty years of age. And if a man of that age ſhould relax a little in his bodily labour, after having

worked hard till that time, it is no more than nature and reaſon will warrant him in doing, and ought not to be cenſured or lamented. Total idleneſs is indeed bad at any age; and we may truſt to his long and rooted habits of induſtry for his not falling into that.

Further, I do not imagine that poor perſons will often chuſe to lay out all their money in the purchaſe of theſe remote life-annuities, but only half, or perhaps a third, or fourth part of it, reſerving the reſt for their more immediate occaſions. Thus, I ſuppoſe that a journeyman cabinet-maker of thirty years of age, who ſhall have ſaved by that time a hundred pounds, will lay out only thirty, or five and thirty pounds of it in the purchaſe of a pariſh life-annuity, to commence when he ſhall be ſixty years of age, (for which he may have an annuity of 20l. a year at that age;) and that he will keep the other ſixty-five or ſeventy pounds in his own power, and lay it out in the public funds, or in ſome other way that he ſhall think advantageous, and in which he ſhall be able to get at it; and with this ſixty-five or ſeventy pounds he will endeavour to advance himſelf in his buſineſs of

a cabinet-

a cabinet-maker. Or, perhaps, he will chuſe to buy two life-annuities of the pariſh, of 10l. each, the one to commence when he ſhall be fifty years old, the other when he ſhall be three-ſcore. In this caſe he muſt pay about 40l. for the firſt annuity, and about 16l. for the ſecond; that is, he muſt ſpend 56l. out of his 100l. and reſerve only 44l. for his preſent occaſions. This conduct may not perhaps be quite ſo prudent as the other. But in either caſe, ſuch a journeyman cabinet-maker would be more likely to keep cloſe to his buſineſs, and to marry and bring up a family, that is, to be the reverſe of a drone, than his brother journeyman in the ſame buſineſs, who ſhall either have worked fewer days in the week than he has done, and ſo have earned leſs money, or having worked as much and earned as much money, ſhall have ſpent it, as faſt as he got it, in the indulgence of preſent pleaſure; nay, even in my opinion, than he himſelf would have been, if, inſtead of purchaſing theſe remote annuities, he had continued in poſſeſſion of his whole hundred pounds; ſince the certainty of not being diſtreſſed in his old age has a natural tendency to make him bolder during the preſent

time, and leſs afraid to venture into ſituations of hazard and difficulty. And when ſuch a man had married, or, if he was already married, it is probable that his next object would be to ſave another 30 or 40 pounds out of his gettings, in order to purchaſe ſuch another annuity of 20l. a year for his wife, to commence in her latter age; after which he would be the better able to employ the remaining part of what he could acquire in the maintenance and education of his children and the portioning and advancing them in the world.

In ſhort, I appeal to every man of property in the kingdom who has reflected upon this ſubject, whether he does not wiſh to ſee every poor perſon he is connected with and concerned for, ſave as much money as they can out of their yearly wages or other profits, in order to provide for themſelves in their old age, when their health and ſtrength ſhall fail them. Many gentlemen of fortune, to encourage their own ſervants to do ſo, borrow their money of them at 4, and ſometimes 5 per cent, though they either have no need of it themſelves, or could borrow it cheaper elſewhere. The deſign of the

the preſent bill is to enable pariſhes to do the ſame good office to their reſpective poor; and from the ſame motive, namely, to encourage them to induſtry and frugality. And this the preſent propoſal does in a more powerful manner than ſuch a beneficent gentleman as is above-mentioned can do, by only borrowing this money at intereſt; that is, in a manner more likely to ſtrike the imaginations of the poor objects of it, and tempt them to employ their money in this way; namely, by ſetting before them the proſpect of a greater annual income for their money than can be had by merely putting it out to intereſt even at 5 per cent. For, if the annuities they purchaſe are to begin in five or ſix years time, they will uſually get nine or ten per cent for their money; and, if they are to begin at the diſtance of 25 or 30 years, not leſs than thirty or forty per cent. What therefore is ſo much approved and encouraged in private families and particular caſes, cannot ſurely be wrong when extended to a greater number of the like objects by means of a public law.

OB-

OBJECTION VIII. and laſt.

Will not loſſes frequently happen to pariſhes by bad agents, and in ſpight of all imaginable care of whom they truſt?

ANSWER.

Perhaps ſuch loſſes may ſometimes happen; but, I ſhould hope, not frequently; as the agents, or attornies, whom the pariſhes would employ to manage their ſtock in the 3 per cent bank-annuities, would probably be, for the moſt part, bankers in London, or ſtock-brokers of good ſubſtance and character; and ſometimes, perhaps, rich merchants, or other gentlemen, reſiding in London, who would undertake this buſineſs from motives of charity and public ſpirit. However, it muſt be confeſſed that this is the weak ſide of the project: and nothing but experience can inform us how far it is practicable for the pariſhes to get their reſpective ſtocks in the bank-annuities managed with fidelity and diligence. And therefore to prevent any pariſhes from being too deeply involved in this ſcheme, who ſhall think at firſt, or afterwards upon trial come to find, that they

are unequal to the taſk of conducting it properly, it is provided in the preſent bill, in the firſt place, that no pariſh ſhall be bound by it till they ſhall have well conſidered and adopted it in a public meeting of ſuch of its inhabitants as ar liable to be rated to the poor's rate; and ſecondly, that, after they ſhall have thus adopted it, and carried it into execution for ſome years, they ſhall be at liberty to conſider it again in another public meeting of the ſaid rateable inhabitants, and, (if they ſhall then, upon the trial they will have had of it, judge it to be inconvenient,) to deſiſt from the further execution of it, and make no more grants of ſuch life-annuities ever after; but ſhall only be obliged to make good the few ſuch annuities they may happen to have granted during the interval in which they had carried it into execution. And, with theſe two optional clauſes, I think the preſent bill may be juſtly conſidered as a ſafe experiment of a ſcheme that *may* prove of very great and publick benefit.

POST-

POSTSCRIPT.

HAving now done with the ſubject of pariſh annuities, I ſhall here ſubjoin the ſolution of another problem in the doctrine of life-annuities, to which the contemplation of that ſubject, and the ſolution of the two former problems, hath inſenſibly led me, and which will probably afford ſome amuſement to ſuch of my Readers as are fond of this doctrine, and of ſeeing it applied to ſubjects of a public, or political, nature. This problem is as follows.

PROBLEM III.

Suppoſe, a man of thirty years of age ſhould be willing to purchaſe an annuity of one pound a year, that ſhould commence immediately, that is, ſo that the firſt payment thereof ſhall be made at the end of a year, or when he will be thirty-one years old, if he ſhall live ſo long; and that ſhould continue for the ſpace

of thirty years certain, ſo that, if he ſhould die in the mean time, it ſhould be paid to his executor, or adminiſtrator; and ſhould continue farther for his life, in caſe he ſhould live beyond the ſaid thirty years. And ſuppoſe the intereſt of money to be $3\frac{1}{2}$ per cent; and the probabilities of the duration of human life at different ages to be ſuch as they are repreſented to be in the table given of them by Dr. Price in his *Obſervations on Reverſionary Payments*, p. 317. table iv; and the values of an annuity of 1l. for the lives of perſons of different ages to be ſuch as they are repreſented to be in the ſame book of Dr. Price, pages 319, 320, 321, table vi. It is required to find what price ſuch a purchaſer ought to pay for the ſaid annuity of one pound a year for the ſaid term of thirty years certain, and further during his own life.

The right or intereſt to be purchaſed in this annuity conſiſts of two parts; a part that is certain, namely, the intereſt in it for thirty years; (it being to be paid either to him, or his aſſigns, or other repreſentatives, during that term of years, whether he lives ſo long, or not) and a part that

that is uncertain or contingent, namely, the intereſt in it for his own life after the expiration of the ſaid term of thirty years, in caſe he ſhall live beyond it. Theſe intereſts muſt be ſeparately computed.

Now, by table iid, page 313, of Dr. Price's book above-mentioned, it appears that the value of an annuity of one pound a year for thirty years certain, when the intereſt of money is 3½ per cent, is 18.3920l. or 18l. 7s. 10d.

And the value of an annuity of 1l. a year for the life of a man of thirty years of age, to commence at the diſtance of thirty years, or when he ſhall be ſixty years of age, that is, ſo that the firſt payment of it ſhall be made when he is ſixty-one years old, may be computed as follows, upon the principles laid down in the foregoing, or ſecond, problem.

An immediate annuity of 1l. per annum for the life of a man of ſixty, upon the ſuppoſitions of this problem, is worth 9.3611. as appears by Dr. Price's book, table vi. page 320. Therefore if the purchaſer was already ſixty years old, he

ought to pay 9.3611, for this annuity. But as he is only thirty years old, and the money he pays for the annuity will, or may, increaſe in the hands of the perſon to whom it is paid, and who grants the annuity, by being improved at compound intereſt for the ſaid ſpace of thirty years, it is evident that it ought to be leſs than 9.3611. even if the purchaſer was ſure of living to the age of ſixty. And in that caſe it ought only to be ſuch a ſum as, being ſo improved during that term, would amount at the end of it to 9.3611. Now it appears by table 1ſt in Dr. Price's book abovementioned, p. 309, that, when the intereſt of money is $3\frac{1}{2}$ per cent, .3562781. or $\frac{3562781}{10000000}$ of a pound, will in the courſe of 30 years grow up to a whole pound. Therefore 9.361 times .3562781. that is, 3.3351. will in the ſame time grow up to 9.361 times a whole pound, or 9.3611. Therefore 3.3351. or 3l. 6s. 8d. $\frac{1}{2}$ is the ſum which the purchaſer of thirty years of age ought to give for an annuity of 1l. a year for his life, to commence when he ſhould be ſixty years, if he was ſure of living to that age.

But as this is not the caſe, it is evident that we muſt again diminiſh the ſum to be paid by

by the ſaid purchaſer of thirty years of age for the remote annuity of one pound, which is to commence only when he is ſixty, on account of the chance of his dying before that age. Now, by the Northampton table of the probabilities of human life in Dr. Price's book aforeſaid, page 317, it appears that of 435 perſons of thirty years of age, all living at the ſame time, only 203 will reach the age of ſixty. Therefore, if the grantor of the annuity of one pound was to grant 435 ſuch annuities to as many different perſons all of thirty years of age, to commence at the diſtance of thirty years, he would only have 203 of them to pay at the end of the ſaid thirty years. Therefore he ought to have only 203 times 9.3611. in his hands, at the end of the ſaid thirty years, ariſing from the contributions of the ſeveral purchaſers of theſe annuities, to pay them with, in order that he may neither get nor loſe by the ſum total of all his bargains; that being the value of each of the annuities which he would then have to pay. Therefore all the 435 purchaſers of theſe annuities (inſtead of contributing 3.3351. or 3l. 6s. 8d. ½ each, when they are thirty years of age, as they

muſt

must have done if they had all been sure of living to sixty, in order to raise a sum in the hands of the grantor of 435 times 9.361l. which would in that case have been wanted to enable him to pay all their annuities) will in the present case be bound only to pay him such lesser sum than 3.335l. or 3l. 6s. 8d. ½ each, at the said age of thirty years, as will, by being improved at compound interest at 3½ per cent, produce at the end of the said thirty years the sum of 203 times 9.361l. or 1900.283l. Now since .3562781. or $\frac{3\,5\,6\,2\,7\,8}{1\,0\,0\,0\,0\,0\,0}$ of a pound, when the interest of money is 3½ per cent, will in the course of thirty years grow up to a whole pound, it follows that 1900.283 times .356.2781. will in the same time grow up to 1900.283 times a whole pound, or 1900.283l. that is, 676.9291. will in the course of thirty years grow up to 1900.283l. Therefore 676.9291. is the sum total of all the contributions of these 435 purchasers of these annuities. Therefore the 435th part of 676.9291. that is, 1.579l. or 1l. 11s. 7d. is the sum which each of the said purchasers of these annuities ought to pay at the age of thirty years for an annuity of one pound for his life, to com-

commence when he is ſixty years of age, and whereof the firſt payment is to be made when he is ſixty-one. Conſequently this is the fair price of ſuch an annuity for a ſingle purchaſer, where only one ſuch bargain is made, unleſs there is ſome reaſon to think ſuch purchaſer to be either more or leſs likely to live than the common run of mankind.

Add therefore this ſum of 1l. 11s. 7d. to 18l. 7s. 10d. which is the value of an annuity of 1l. a year for thirty years certain; and the ſum total thereby obtained, to wit, 19l. 19s. 5d. will be the price which the purchaſer of thirty years of age ought to pay for an annuity of 1l. for thirty years certain, and further for the term of his own life.

Coroll. 1. When the 3 per cent bank-annuities ſell for 85.7142l. or 85l. 14s. 3d. ½ per 100l. ſtock, every hundred pounds of ſterling money laid out in the purchaſe of thoſe annuities will produce an intereſt of 3½l. or 3l. 10s. per annum. For 3½ is to 100 as 3 is to 85.7142.

Therefore

Therefore when the 3 per cent bank-annuities ſell for 87l. or 86½ l. per 100l. ſtock, the money laid out in the purchaſe of them will produce ſomewhat leſs than 3½l. or 3l. 10s. per annum for every hundred pounds. This is the caſe at preſent, January, 1773. We may therefore reckon, that, if the Government wanted to borrow a moderate ſum of money of the people, they could borrow it at the rate of 3½ per cent, or at rather a lower intereſt. But the difference is ſo ſmall that it does not ſeem worth attending to.

Suppoſe therefore, that it was neceſſary to borrow a million of pounds ſterling of the people, and that the parliament, through an unwillingneſs to burthen the nation with a perpetual tax for the payment of the intereſt of it, ſhould, (inſtead of granting the lenders of it a perpetual annuity of 3l. 10s. or thereabouts, for every 100l. they advanced, as is the common way,) reſolve to grant annuities to them for thirty years certain, and for the further term of their own lives reſpectively; and ſhould direct, that no perſon under thirty years of age ſhould become a purchaſer of any ſuch annuity

nuity for his own life. In ſuch a caſe the Government would be intitled to receive for every annuity of one pound a year, which it ſo granted, the ſum of 19l. 19s. 5d. and conſequently for every annuity of five pounds a year ſo granted the ſum of five times 19l. 19s. 5d. or 99l. 17s. 1d. or nearly 100l. that is, the Government might raiſe money in this way at about five per cent. and be rid of great part of the burthen at the end of thirty years. For at the end of the thirty years every 435 of the contributors, or purchaſers of theſe annuities, would be reduced to 203: and the executors, or other repreſentatives of the deceaſed 232 contributors, would have no further claim upon Government: ſo that the annual charge upon the nation would be reduced at once to leſs than half. And this remaining burthen of the annuities that would be due to the ſurviving contributors for their lives, would evidently be decreaſing every year, and would in thirteen years more be reduced to one half, and in another ſpace of ſeven years to leſs than half of the laſt half, or an eighth part of the firſt annual charge. For the 203 contributors that would remain alive at the end of the

thirty years, out of every original 435, would in thirteen years more be reduced to 99, and in ſeven years more to 46.

Coroll. 2. If, inſtead of an annuity of five pounds a year for a term of thirty years certain, and for the further term of the lender's life, (as is ſuppoſed in the foregoing corollary) the Government were to offer the lenders of the money an annuity of 5l. a year for only 20 years certain, to be paid either to themſelves or their executors, or other perſonal repreſentatives; and after the ſaid 20 years, a right to the ſhares of the deceaſed contributors by ſurvivorſhip for 10 years more, and then at the end of the ſaid 10 years, a right to the original annuity of 5l. a year for their reſpective lives; it is evident, that the price to be paid to government for ſuch an annuity, ought to be the ſame as in the foregoing corollary, to wit, 100l. for every annuity of 5l. a year ſo granted. For the government would pay the ſame ſums of money every year to the lenders of the money, or their repreſentatives, and for the ſame number of years, on this ſuppoſition as on the former. Only in this caſe

caſe it would be wholly enjoyed by the ſurviving contributors after the expiration of twenty years: whereas in the former caſe it would be enjoyed for the next ten years by both the ſurviving contributors and the repreſentatives of thoſe that were deceaſed.

Though this propoſal, and the former, are in themſelves equally advantageous to the contributors, yet it is not unlikely that many of them would give the preference to the latter, from a hope that they might live to enjoy the increaſe of their annuities by the deaths of their companions during the laſt 10 years of the 30. Yet there is one circumſtance in it, which perhaps would not thoroughly pleaſe them. This is the reduction that would take place at the end of the thirty years in their reſpective annuities from the ſums to which they would have been increaſed by ſurvivorſhip during the laſt ten years of the thirty, to their original quantity, that is, from $\frac{435}{203} \times 5$l. or 2.142×5l. or 10.710l. or 10l. 14s. 2d. $\frac{1}{2}$ to 5l. And it is probable they might wiſh to continue to receive the increaſed annuity of 10l. 14s. 2d. $\frac{1}{2}$ during their lives. But in this caſe they muſt

pay a greater ſum than 100l. for an annuity of 5l. The computation of the additional price neceſſary to be paid on this account, ſhall be the ſubject of the next corollary.

Coroll. 3. Suppoſe therefore that the government ſhould propoſe to grant annuities of 5l. each for 20 years certain, and further for 10 years more with benefit of ſurvivorſhip, and afterwards to continue to the ſurviving contributors of the money raiſed in this method, the enjoyment of their increaſed annuities, as they ſtood at the end of the 30 years, during their lives reſpectively. It is required to determine, what ſum of money a contributor ought to pay for every ſuch annuity of five pounds.

Now by the laſt corollary it appears, that if each contributor was to enjoy an annuity of 5l. during 20 years certain, ſo that, if he died in the mean time, his executor, or other repreſentative, ſhould receive it during that term; and afterwards, if he lived beyond that term, was to enjoy the ſame, with the benefit of ſurvivorſhip, for 10 years more; and then to

enjoy

enjoy his firſt annuity of five pounds during the remainder of his life; he ought to pay for the ſaid annuity the ſum of 99l. 17s. 1d. He muſt therefore pay for the ſame annuity of five pounds, with the additional advantage here propoſed, ſuch ſum of money over and above the ſaid ſum of 99l. 17s. 1d. as is the juſt value of an annuity for his life, to commence at the end of the ſaid thirty years, that is equal to the difference of the ſaid original annuity of 5 pounds, and the increaſed annuity of 10l. 14s. 2d. $\frac{1}{2}$, or 10.710l. which would be the annuity that he would receive at the end of the ſaid 30 years, or in the 31ſt year, if the right of ſurvivorſhip had been continued ſo far; that is, ſuch ſum of money as is the juſt value of an annuity of 5l. 14s. 2d. $\frac{1}{2}$, or 5.710l. for his life, to commence at the end of the ſaid thirty years. Now, ſince by the computation in this problem it appears that an annuity of 1l. a year, for the life of a man of thirty years of age, to commence when he ſhall be ſixty years old, ſo that the firſt payment thereof ſhall be made when he is ſixty-one years old, is worth 1l. 11s. 7d. or 1.579l. it follows, that an annuity of 5.710l. for his life, to commence

mence at the ſame time, will be worth 5.710 times 1.579l. or 9.016l. or 9l. os. 3d. $\frac{3}{4}$, or, nearly, 9l. os. 4d. Therefore 9l. os. 4d. muſt be added to 99l. 17s. 1d.; and the ſum thereby obtained, to wit, 108l. 17s. 5d. will be the price which the contributor ought to pay to the Government for an annuity of 5 pounds attended with the forementioned advantages.

QE J.

Coroll. 4. Therefore for a contribution of only 100l. the contributor would have a right to an annuity of only 4.592l. or 4l. 11s. 10d. a year attended with the advantages deſcribed in the laſt corollary. For 108l. 17s. 5d. or 108.871l. is to 100l. as 5l. is to 4.592l. or 4l. 11s. 10d.

Coroll. 5. And for a contribution of only 87l. the contributor would have a right to an annuity of only 3.995l. or 3l. 19s. 10d. $\frac{3}{4}$ a year, attended with the advantages deſcribed in the third corollary. For 108l. 17s. 5d. or 108.871l. is to 87l. as 5l. is to 3.995l. or 3l. 19s. 10d. $\frac{3}{4}$

Coroll.

COROLL. 6. It follows from the laſt corollary, that, if a million of the 3 per cent bank-annuities, (which now ſell at about 87l. per cent) were (with the conſent of the ſeveral proprietors of them) to be converted into annuities of the laſt-mentioned kind, that is, for a term certain of twenty years, and then for ten years more with the benefit of ſurvivorſhip, and then with a continuation of the ſaid annuities, at the quantities to which they would have increaſed during the ſaid ten years, for the lives of the reſpective proprietors, and the ſaid proprietors were all thirty years of age; they would be intitled to an annuity of 3l. 19s. 10d. $\frac{3}{4}$, or, very nearly, to an annuity of 4 pounds a year, attended with theſe advantages, for every 100l. ſtock they were poſſeſſed of.

Therefore if ten millions of the ſaid 3 per cent bank-annuities, (the intereſt of which is 300,000l. a year) were to be converted, with the conſent of the proprietors of them, into annuities of this kind, the intereſt neceſſary to pay them would be 400,000l. a year inſtead of 300,000l.; that is, an additional ſum of 100,000l.

100,000l. a year muſt be appropriated to this purpoſe. And if fifty millions of the ſaid 3 per cent bank-annuities were to be converted in like manner into annuities of this kind, the additional intereſt to be paid to the proprietors would be 500,000l. or half a million, a year; which muſt be taken out of the ſinking fund, and appropriated to this purpoſe. And, if 100 millions of the ſaid 3 per cent bank-annuities, (or a quantity of the public funds compoſed of different ſorts of annuities, but equivalent to 100 millions of the ſaid 3 per cent bank-annuities) were in like manner to be converted into annuities of this kind, the additional intereſt to be paid to the proprietors would be a whole million a year; which muſt therefore be taken out of the ſinking fund and appropriated to this purpoſe. And the money thus appropriated to the payment of this additional intereſt would be put out of the reach of even the parliament itſelf, and could never be diverted by them during the ſaid term of thirty years to any other purpoſe whatſoever; which ſtrict appropriation of the ſinking fund is what Dr. Price, and many other judicious perſons and lovers of their country, ſo earneſtly recommend

commend and wiſh for, as the only method of preventing a national bankruptcy. Sir John Barnard in particular, whoſe opinion ought to have great weight upon this ſubject, is reported to have often recommended to parliament this ſtrict application of the ſinking fund to the purpoſe of its original deſtination, the diſcharging the national debt. *At leaſt*, was he uſed to ſay, *let me ſee half a million ſo applied, and ſet apart for this ſingle purpoſe, and conſidered as ſacred to that alone, and incapable of being diverted, upon any occaſion or under any pretence whatſoever, to any other purpoſe; and in a moderate number of years it will do great things towards obtaining this important end.* Since Sir John Barnard's time the national debt has been nearly doubled, having increaſed from 70 to near 140 millions, and now is pretty near 130 millions. And, fortunately for the nation, the ſinking fund is nearly doubled likewiſe; having amounted of late years to about two millions of pounds a year, one year with another. It ſeems therefore to be as reaſonable to appropriate at this day a whole million of pounds out of this fund to the conſtant diſcharge of the public debt in the manner recommended

by that celebrated patriot, as it was in his time to appropriate half a million to the ſame purpoſe. And the converſion of the perpetual annuities into ſuch annuities for a moderate number of years, as are above deſcribed, with a continuance of them for the reſpective lives of the proprietors, in order to induce them to conſent to ſuch a change, ſeems to be the moſt effectual method that can be taken to ſecure the conſtant application of ſuch appropriated part of the ſinking fund to the purpoſes of its deſtination. For then it cannot be diverted to any other uſes without an abſolute breach of the national faith, which, I doubt not, all miniſters and parliaments will always conſider as a moral impoſſibility.

If one million of pounds *per annum* out of the ſinking fund was ſo appropriated, and the other million was uſually applied to the ſame great purpoſe of diſcharging the national debt, and never diverted to any other purpoſes, except on very urgent and important occaſions, the whole national debt might probably be diſcharged, or reduced to a very moderate quantity, as for inſtance, to 20 millions of pounds,

(which

(which ſome people think would be rather better than to diſcharge the whole of it,) in little more than thirty years, provided we continue ſo long to enjoy the bleſſings of peace, and trade, and liberty, in the degree we do at preſent.

The foregoing problems have been ſolved at great length, not only with a view to make them as plain and intelligible as poſſible to perſons who are not uſed to enquiries of this nature, but likewiſe to guard againſt any erroneous reaſonings in the ſolution of them, and to make it eaſy for any one to examine the grounds and principles upon which they have been determined, and detect any fallacy that may happen to lurk amongſt them. For the doctrine of chances and annuities is a ſubject of ſo ſubtle a nature, that one cannot be too careful to avoid falling into ſuch miſtakes. For even Mr. De Moivre himſelf, who was ſo juſtly celebrated for his extenſive knowledge of this doctrine, has been found not totally free from them. One of theſe miſtakes, in a queſtion concerning a reverſion depending on a ſurvivorſhip, has been mentioned by Dr.

Price in his *Treatiſe on Reverſionary Payments*, Eſſay iii. pages 228, 229, &c. and properly corrected by him. But, conſidering the great authority of Mr. De Moivre, it will not, I imagine, be thought ſuperfluous to add another ſolution of the queſtion; which will ſerve as a further proof of this miſtake, and a confirmation of Dr. Price's correction. The queſtion propoſed by Mr. De Moivre, as it is ſtated in Dr. Price's book, is as follows.

QUESTION.

"A. aged 40, expects to come to the poſ-
"ſeſſion of an eſtate, if he ſhould ſurvive B.
"aged likewiſe 40. In theſe circumſtances he
"offers, in order to raiſe a preſent ſum, to
"give ſecurity for 40l. *per annum* for ever,
"out of the eſtate at his death, provided he
"ſhould get into poſſeſſion, that is, provided
"he ſhould ſurvive B. What is the ſum that
"ought now to be advanced to him in conſi-
"deration of ſuch ſecurity, reckoning com-
"pound intereſt at 4 per cent."

Mr. De Moivre's directions in his treatiſe on Annuities, problems 17 and 20, lead us to ſeek

ſeek the required ſum in this caſe by the following proceſs.

Find firſt the preſent ſum which A. ſhould receive for the reverſion of 40l. per annum for ever after his death, ſuppoſing it not dependent on his ſurviving B. The preſent value of ſuch a reverſion is found by ſubtracting the value of the annuity of 40l. for the life of A. from the value of the perpetuity of it. Now the value of an annuity of 1l. for the life of a man of 40 years of age, when the intereſt of money is 4 per cent. is 13.196l. Therefore the value of an annuity of 40l. for the ſame life, is 40 times 13.196l. or 527.840l. And the value of a perpetual annuity of 40l. when the intereſt of mony is 4 per cent. is 25 times 40l. or 1000l. Therefore the value of the reverſion of the annuity of 40l. for ever after the death of A. is the exceſs of 1000l. above 527.840l. that is, 472.160l. or 472l. 3s. 2d.½ But, as Mr. De Moivre obſerves, the lender having a chance to loſe his money, a compenſation ought to be made to him for the riſk he runs, which is founded on the poſſibility that a man of forty years of age may not

not ſurvive another perſon of the ſame age. This chance is an equal chance; and therefore half the preceding ſum, or 236l. 1s. 7d.$\frac{1}{4}$, is the ſum which ſhould be paid for the reverſion of the ſaid annuity of 40l. a year after the death of A, in caſe he ſhall out-live B.

This is the ſolution which Mr. De Moivre gives of this queſtion, and which Dr. Price ſhews to be erroneous in the following pages of that eſſay. And the error is by no means trifling. For the true value of the reverſion in queſtion is only 168l. 12s. This will appear from the ſolution of the following problem, and its corollary.

PROBLEM IV.

Suppoſe a perpetual annuity of 40l. a year, to be ſettled in the hands of truſtees for the uſe of two perſons A and B, each of the age of forty years, during their lives and the life of the longeſt liver of them, to be enjoyed by them in ſucceſſion, one after the other, to wit, firſt by B during his life, and then by A during

during his life; and afterwards to the uſe of C and his heirs and aſſigns for ever; and that the intereſt of money is 4 per cent. It is required to find the value of C's reverſionary intereſt in this annuity.

It is evident that, if from the value of the perpetuity of this annuity, or of the right to the immediate enjoyment and free diſpoſal of it, we ſubtract the value of the right to enjoy it during the lives of A and B, and the life of the longer liver of them, the remainder will be the value of C's reverſionary intereſt in it. This is ſo plain that it neither needs, nor will admit of, any proof. Now the value of an annuity for the lives of two perſons of given ages, and the life of the longer liver of them, is allowed by all the calculators to be equal to the exceſs of the ſum of the values of the ſame annuity for the two ſingle lives above the value of it for the two joint lives. The value of an annuity of 40l. for the ſingle life of either A or B, (they being both 40 years of age) is 40 times 13.196l. or 527.840l. Therefore the ſum of the values of an annuity of 40l. for the two lives of A and B is twice

527.840l.

527.840l. or 1055.680l. And, by table vii. of Dr. Price's book on reverſionary payments, p. 324, the value of an annuity of 40l. for the joint lives of two perſons, each of 40 years of age, is 40 times 9.8261. or 393.040l. which, being ſubtracted from 1055.680l. leaves the ſum of 662.640l. This therefore is the value of an annuity of 40l. for the lives of A and B, and the life of the longer liver of them, and conſequently muſt be ſubtracted from the value of the perpetuity of the ſame annuity of 40l. that is, from 40 times 25l. or from 1000l. and the remaining ſum of 337.360l. or 337l. 7s. 2d.½, will be the true value of C's reverſionary intereſt in the propoſed annuity of 40l. Q E J.

Coroll. 1. Now let C be ſuppoſed to ſell his certain reverſionary right to this annuity of 40l. to two different purchaſers D and E, ſo that, if A ſhould out-live B, the annuity ſhall belong to D after the death of both A and B, and, if B ſhall out-live A, it ſhall belong to E after the death of the ſame perſons. In this caſe it is evident, that C will have parted with his whole intereſt in this annuity, and conſe-

conſequently muſt receive from both the purchaſers together, the whole value of it, that is, the ſum of 337l. 7s. 2d. ½. And ſince each of the purchaſers is to wait till the ſame period before he can come into poſſeſſion of this annuity, in caſe it ſhould fall to his lot, namely, till the death of both A and B; and each of them has the ſame chance of obtaining it by the ſurvivorſhip of the perſon on whom his lot depends, becauſe A and B are ſuppoſed to be both of the ſame age, and conſequently equally likely to live; it follows, that each of them muſt pay to C one half of the ſum of 337l. 7s. 2d. ½. that is, the ſum of 168l. 13s. 7d. ¼. for his contingent intereſt in this annuity.

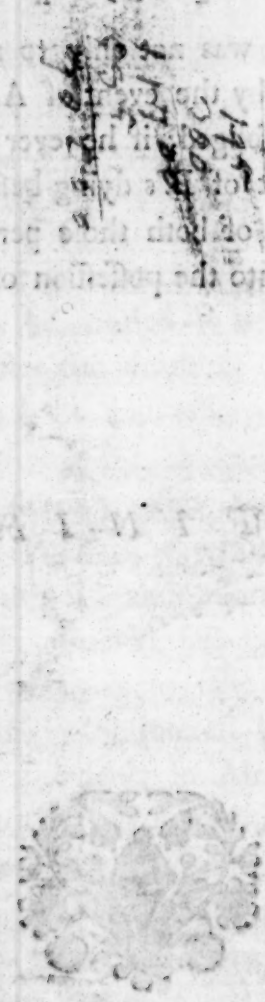
COROLL. 2. Now let D be ſuppoſed to ſell his contingent intereſt in this annuity of 40l. to A himſelf, who hitherto has been ſuppoſed to have a contingent intereſt in it only for his life: and then let A be ſuppoſed to ſell the ſame to F.

It is evident that A ought to give the ſame price for it to D as D gave to C, that is, 168l.

13s. 7d. $\frac{1}{4}$: and F ought to give this price for it to A.

Now theſe ſuppoſitions bring us to the terms of Mr. De Moivre's queſtion above-mentioned. For A will, after the purchaſe of D's intereſt in this annuity, be intitled to it, not only for his own life after the death of B, as before, but alſo to his heirs and aſſigns for ever, in caſe of his out-living B; as he is ſtated to be in Mr. De Moivre's queſtion. And of theſe two contingent intereſts, the one for his life, the other for his heirs and aſſigns, which he will now poſſeſs, he will ſell to F only the latter intereſt, which is to take place at his death. And this is the intereſt he is ſuppoſed to ſell to the purchaſer in Mr. De Moivre's queſtion. Therefore the ſum of money which that purchaſer ought to pay for the contingent annuity of 40l. mentioned in that queſtion, is only 1681. 13s. 7d. $\frac{1}{4}$. and not 2361. 1s. 7d. $\frac{1}{4}$. as Mr. De Moivre makes it.

The occaſion of this miſtake of Mr. De Moivre, was his not conſidering that the perſon who was to purchaſe the contingent annuity

nuity of 40l. was not only to run the riſk of miſſing of it by the event of A.'s dying before B. but was obliged, if he ever got it by the contrary event of B.'s dying before A. to wait till the death of both thoſe perſons before he could come into the poſſeſſion of it.

F I N I S.